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# An application of the polynomial distributed lag to monthly, quarterly, and annual data in the demand function for money

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# An application of the polynomial distributed lag to monthly, quarterly, and annual data in the demand function for money

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# CHAPTER I. INTRODUCTION

This study used the Lagrange Interpolating Polynomial Technique in order to estimate distributed lags in the demand for money function. The study's purpose was to estimate and compare the elasticity coefficients and weight structures associated with the various independent variables using monthly, quarterly, and annual data. It was felt that these comparisons might provide some evidence as to any change or lack of change in the coefficients or weights as a result of the type of data used.

In this chapter a brief review of some of the empirical work concerning the demand for money function is presented. The empirical work is important because it represents attempts to provide evidence on the theoretical issues which surround the demand for money topic. This brief review is presented here in the hope that it will be useful in providing a theoretical basis for the model used in this study. The second chapter is a discussion of the Lagrange Interpolating Polynomial Technique. Chapter III presents the general model used in this study and discusses the problem of serial correlation. Chapter IV contains the results of the study and Chapter V is a summary.

The importance of empirical demand for money models can be illustrated by a quote from Teigen (1970, p. 74):

> To the extent that such models are used as a guide for policy decisions, accurate knowledge concerning the proper form and arguments of these functions, and their elasticities, is crucial for the correct choice of economic policy instruments.

As this quote suggests there are many issues surrounding the demand for money function. A partial listing of some of these issues, and a list of topics to be discussed in this chapter, would include:

1. the proper specification of the way in which interest rates enter the demand for money function
2. the selection of the appropriate interest rate(s) for this function
3. other independent variables to be included in the function
4. the appropriate definition of money
5. single and simultaneous equation models of the demand for money.

Most theoretical work on the demand for money includes an interest rate as an argument in the demand for money function. Keynes (1964) expanded the classical concept of money demand by adding the speculative demand for money to the already existent transactions and precautionary demand. By recognizing that money might be held as an asset, just like bonds, Keynes introduced the interest rate into the money demand function. The role for interest rates was further strengthened by such people as Baumol (1952) and Tobin (1956) when they linked the interest rate to the transactions demand for money.

Friedman (1959), Baumol, Tobin, Keynes and others have argued forcefully for the inclusion of interest rates in the theoretical formulation of money demand. However, Friedman (1959), who agreed on the inclusion of interest rates in the money demand function on theoretical grounds, questioned the empirical validity of incorporating a rate of interest into the demand for money function.

> In our experiments, the rate of interest had an effect in the direction to be expected from theoretical considerations, but too small to be statistically significant. (Friedman, 1959, p. 329)

Friedman's experiments were concerned with predicting the velocity of circulation of money. He estimated a money demand function where real per capita money, broadly defined, was the dependent variable and permanent real per capita income was the independent variable. Using his estimates of the money demand function he then made projections of the velocity of circulation of money. His projections of velocity were quite accurate and the errors which were present in the predictions were not closely related to the level of interest rates. Because of this lack of relation between the prediction errors and interest rate, Friedman concluded that the rate of interest was not statistically significant in explaining variations in money balances.

Laidler (1966b), using the same type of data and an almost identical time period, refitted Friedman's original equation and compared it with one where an interest rate was included. He then used the two equations to predict annual levels of per capita real money balances. The equation which contained the interest rate variable had a lower mean error of prediction than did the equation without interest rates.[1] According to Laidler, Friedman's finding, that the errors in prediction did not appear to be closely related to the interest rate, was due to the fact that he had failed to include an interest rate variable. This omission caused the intercept to fall and the income elasticity to rise in Friedman's money demand formulation. On the basis of his tests Laidler concluded that the rate of interest was significant in the demand function for money.

Given that interest rates should enter the demand for money function, the problem becomes whether these rates should be current or lagged. Laidler's test involved current interest rates. Others, like Hamburger (1966), introduced a lagged, rather than a current interest rate into the money demand equation. Hamburger based his lag on the supposition that people, because of the costs involved in information gathering and the carrying out of transactions, do not change their portfolios immediately. Therefore, people respond to changes

---

[1]The mean error of prediction is the arithmetic average of the absolute value of the difference between the actual and predicted values of a given variable for all points for which a prediction is obtained.

in the market with a lag. The evidence from Hamburger's and other studies indicate that the use of a lagged interest rate, a weighted average of current and past values of the interest rate, provide a better fit for money demand equations than does the use of a current rate.

Whether a current or a lagged rate is used, the theoretical and empirical importance of the interest rate in money demand functions is well established. However, there is no one rate of interest. There are short, intermediate, and long rates. There are rates on debts, rates on equities, explicit rates such as the rate paid on time deposits, implicit rates attributable to physical goods and services, and so on. Since the specification of a money demand function is a theoretical problem, the interest rate or rates used in money demand equations depends upon theoretical considerations. If one takes a portfolio approach to money demand as Brunner and Meltzer (1964) do, rates of return on both debt and equity should be incorporated into the function. Or, still using the portfolio approach, one might follow Dickson and Starleaf (1972) and incorporate the yield on time deposits along with some market determined interest rate.

Explicit inclusion of the rate on time deposits is a recognition of the fact that time deposits are good substitutes for money (narrowly defined) in peoples' portfolios.

Furthermore, to the extent that time deposits are found in peoples' portfolios along with bonds and equities, inclusion of the time deposit rate may be more necessary than inclusion of any of the other specific rates. The reason for this is that if the yield on time deposits is held below market determined rates by a legal ceiling, a market determined rate acting as a vector of rates for many money substitutes would not be a good proxy for the yield on commercial bank time deposits.

There is no consensus on the proper number of yields to use in a money demand equation. There is also no consensus as to whether short or long-term yields are best. Bronfenbrenner and Mayer (1960), concentrating on the closeness of substitutes for money, prefer to use a short-term rate since it measures the opportunity cost of holding money. Latane (1954) and Meltzer (1963) prefer a long-term rate, arguing that in a theory of portfolio selection money should be compared with the highest yielding asset. Eisner (1963) also favored a long-term rate. He did so because of the importance attached to this rate in a Keynesian system for determining the level of investment and hence, influencing the overall level of economic activity.

Lee (1967) took a different approach and concentrated on differentials between interest rates rather than on absolute levels of rates. He fitted money demand equations which incorporated a variable measuring the differential between the yield on savings and loan association shares and the yield on money, the differential between time deposits and money, between dividend yields and money, and between money and both short and long-term yields on debt. He concluded that the differential that worked the best was between money and savings and loan shares, and therefore, that the demand for money is dependent on interest rates of nonbank intermediary liabilities.

Some studies have compared the short to the long-term rate of interest. Heller (1965) found that the short-term rate as approximated by the yield on sixty to ninety day commercial paper was statistically significant at the five per cent level in all but one of the regression equations designed to estimate the demand for money. The long-term rate as approximated by the yield on government bonds was never significant. On this basis Heller concluded that the short-term rate of interest was a better explanatory variable in the demand function for money than was a long-term rate. Laidler (1966b) also found the short-term rate superior to the long-term rate where the proxies for the short and long rate were the yield on four to six month commercial paper and the yield on twenty year corporate bonds.

Accurate comparisons of empirical studies designed to find the best interest rate or rates are quite hopeless. The time periods differ and the proxies used for the dependent and independent variables change from study to study. Therefore, it appears that there is no conclusive evidence on the best interest rate or rates to be used in money demand studies. The determination of a best rate or rates depends upon both theoretical specification and empirical results. All that can be said is that many different interest rates and combinations of interest rates have been used and proven successful in formulating money demand functions.

Once the question of interest rates has been resolved, the role of other independent variables in the demand function for money must be examined. Primarily the question of other variables refers to whether the appropriate constraint imposed on money balances should be current income, permanent income, wealth, or some combination of these variables. When current income is used as the constraint on the demand for money balances, the role of money as a medium of exchange or to effect a given level of transactions is being emphasized. When permanent income or some measure of wealth is used as the constraint, the demand for money becomes a part of the larger problem of the demand for assets, both financial and physical. And as such, money is viewed as only one way, although an important one, of holding assets. This portfolio approach to the demand for money concentrates on wealth and the yield on assets while down playing the role of current income.

In tests of money demand equations by Brunner and Meltzer (1963) and Meltzer (1963) comparing the statistical significance of the coefficients attached to current income and a measure of wealth, it was found that wealth was superior. Heller (1965) took exception to Brunner and Meltzer's findings. He found that the significance of the income and wealth variables was dependent on the definition of money that was used. In regressions where the narrow definition of money was used as the dependent variable, income was statistically significant and wealth was not. Where the broad definition of money was used, the results were reversed.

Laidler (1966a) and Chow (1966) tested the importance of current income, permanent income, and a measure of wealth in the money demand function. They found permanent income to be a better explanatory variable than either current income or wealth. Once again, as with the interest rate, the empirical evidence concerning the proper constraint for money balances is not conclusive.

In addition to measures of interest rates and income or wealth, a variable representing prices is often included in money demand functions. Studies by Meltzer (1963) and Dickson and Starleaf (1972) indicate that price is a significant variable in explaining the demand for money and further, that the coefficient attached to the price variable demonstrates an absence of money illusion.

Given the problem of clearly delineating what is money from what is not money, it is not surprising that disagreement exists as to the proper dependent variable in money demand studies. The main disagreement is between the use of money narrowly defined ($M_1$ = Currency plus demand deposits) and money broadly defined ($M_2$ = Currency plus demand deposits plus time deposits at commercial banks). Meltzer (1963) suggests that the problem should be resolved by defining money in such a way that the demand function for money remains stable over time in the face of changing economic, political, and social conditions. What is meant by stability of the money demand function can probably best be seen in this quote from Laidler.

> A 'more stable demand for money function' may be taken to be one that requires knowledge of fewer variables and their parameters in order to predict the demand for money with a given degree of accuracy, or, which amounts to the same thing, one that yields parameter estimates that are less subject to variation when the same arguments are included in the function and hence enables more accurate predictions of the demand for money to be made. (Laidler, 1969, p. 516)

The stability of money demand is important because of macroeconomic policy considerations. If there are unpredictable shifts in the demand for money, the effects of such monetary policy actions as changing the money stock will be uncertain. However, the effects of monetary policy will be predictable and have a greater likelihood of success if the money demand function is stable.

Much of the work that has been carried out in an attempt to define money empirically has concerned itself with the degree of substitutability between financial assets. Feige (1964) attempted to measure, on the basis of cross elasticities of demand, the degree of substitutability between assets. He found little relation between such assets as demand and time deposits or between demand deposits and savings and loan shares. On the basis of his evidence, Feige concluded that the narrow definition of money is the best one to use for money demand studies. Feige's results were questioned by Lee (1966). His studies showed that savings and loan shares were a close substitute for money narrowly defined. Laidler (1966a) found the most stable demand function to be the one that used the broad definition of money. Friedman (1959) also used the broad definition, basing his choice on the argument that time deposits are such close substitutes for money that less error is introduced by including them rather than excluding them. Brunner and Meltzer (1963) found the narrow definition best. Heller (1965) felt that either definition could be used as long as the constraint on money balances was modified accordingly.

The empirical studies cited indicate a lack of agreement as to the proper definition of money. Of course the various tests performed to determine whether the narrow or broad definition of money is best are not comparable. They all use different time periods and independent variables. However, they do seem to indicate that either the $M_1$ or $M_2$ definition of money may be appropriate depending on the model tested.

The final issue that will be discussed is the use of single versus simultaneous equation models of the demand for money. Single equation models will yield estimated coefficients that are unbiased, efficient, and so forth, if there is a one-way chain of causation running from the independent variable to the dependent variable with no direct feedback. Therefore, independent variables such as income, wealth, interest rates, and prices must be assumed to influence money demand, but the demand for money must not influence these variables.

In reality we know that the chain of causation runs both ways. Simultaneous interaction between the supply of and demand for money leads to what is termed the identification problem.[1] Basically the problem is in knowing that the function we are estimating is a demand curve. It could be a supply curve or something in between.

This simultaneous equation bias can be illustrated by reference to the estimation procedure of any demand or supply curve. Time series observations of price and quantity used to estimate these curves are not the observations associated

[1] Christ (1966, p. 300).

with any one particular curve. Rather, they represent a series of equilibrium points which change because of the continual shifting of the supply and demand functions. Attempts to derive estimates of these curves on the basis of a single equation model result in estimates which are neither supply or demand curves. In order to get around the identification problem, a simultaneous equation model including both a supply and a demand equation must be used.

In the case of money demand studies, what must be done is to construct a system of simultaneous equations containing both a supply and a demand function for money. Or, more generally, the system must include both a monetary and a real sector of the economy. The coefficients of both functions are then estimated jointly, taking account of the interdependence of the functions.

A model by Teigen (1964) integrated money demand and supply and showed the interrelationships between interest rates and the supply of money. However, because investment was treated as an exogenous variable, the model stopped short of integrating the real and monetary sectors of the economy which would have shown the interaction of not only money and the rate of interest, but also of income with these variables. A study by Brunner and Meltzer (1964) did integrate the monetary and real sector in order to capture more of the interaction between variables. In general, the elasticity coefficients derived from the simultaneous equation models are quite similar to those found with the single equation approach. This gives support to the results of the single equation studies and suggests that the identification problem in these studies may not be serious enough to require the use of a system of simultaneous equations in order to obtain estimates of elasticity coefficients.

# CHAPTER II. THE POLYNOMIAL LAG

This chapter involves the use of lagged values for some of the explanatory variables. The method employed in this study to carry out the estimation procedure using lags will be explained in this chapter. That method is the Lagrangian Interpolating Polynomial Technique as suggested by Almon (1965) as a way to deal with distributed lag equations.

Let us begin by assuming that we have the following money demand function which we would like to estimate using time series data:

$$M_t^d = \alpha_0 + \alpha_1 \; Y_t^* + U_t \tag{2.1}$$

where

$M_t^d$ = the demand for nominal money balances.

$$Y_t^* = \sum_{i=0}^{n} w(i) \; Y_{t-i} \tag{2.2}$$

$U_t$ = the error term

$Y_{t-i}$ = income in period t-i

and we further stipulate that

1) $\sum_{i=0}^{n} w(i) = 1$

2) $w(i) = 0, \; i \geq n$

Substituting 2.2 into 2.1 yields

$$M_t^d = \alpha_0 + \alpha_1\ w(0)\ Y_t + \alpha_1\ w(1)\ Y_{t-1} + \ldots + \alpha_1 w(n)\ Y_{t-n} + U_t \qquad (2.3)$$

The problem is to fit Equation 2.3 and obtain estimates for the $\alpha$'s and the $w(i)$.

Almon's technique assumes that the weights $w(i)$, and hence the $\alpha_1 w(i)$, can be approximated by the values of a polynomial function. The technique is based on Weierstrass's Theorem which states that:

> ...a function continuous in a closed interval can be approximated over the whole interval by a polynomial of suitable degree which differs from the function by less than any given positive quantity at every point of the interval. (Johnston, 1972, p. 294)

Suppose we are given a real function f defined for all x in the interval [0, n], 0<n. According to Weierstrass's Theorem, this function can be approximated by a unique polynomial of degree $\leq n$, called the interpolating polynomial.

In order to obtain the Lagrangian interpolating polynomial, a polynomial

$$P(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n \qquad (2.4)$$

must be found with the property that

$$P(x_i) = f(x_i) \text{ for } i=0,\ 1, \ldots\ n$$

Let $x_0$, $x_1$, ... $x_n$ be n+1 distinct points in the interval [0, n] with f(x) given at these points. The basic idea behind the Lagrange method is to first, find a polynomial which takes on the value one at a particular sample point and second, the value zero at all other sample points.[1] Or

$$L_i(x_j) = 0 \qquad i \neq j \qquad j = 0, 1, \ldots n$$

$$L_i(x_i) = 1$$

The first property indicates that the polynomial has n roots $x_0$, $x_1$, ... $x_n$. A polynomial with those roots must be of the form

$$L_i(x) = c(x-x_0)(x-x_1)\ldots(x-x_{i-1})(x-x_{i+1})\ldots(x-x_n) \qquad (2.5)$$

where c is selected so that $L_i(x_i) = 1$.

Now if

$$L_i(x_i) = c(x_i-x_0)(x_i-x_1)\ldots(x_i-x_{i-1})(x_i-x_{i+1})\ldots(x_i-x_n) = 1 \qquad (2.6)$$

then

$$c = \frac{1}{(x_i-x_0)(x_i-x_1)\ldots(x_i-x_{i-1})(x_i-x_{i+1})\ldots(x_i-x_n)}$$

---

[1] Moursund and Duris (1967, p. 109).

so that the final result is

$$L_i(x) = \frac{(x-x_0)(x-x_1)\ldots(x-x_{i-1})(x-x_{i+1})\ldots(x-x_n)}{(x_i-x_0)(x_i-x_1)\ldots(x_i-x_{i-1})(x_i-x_{i+1})\ldots(x_i-x_n)}$$

or simplifying

$$= \frac{\prod_{\substack{j=0 \\ j\neq i}}^{n} (x-x_j)}{\prod_{\substack{j=0 \\ j\neq i}}^{n} (x_i-x_j)} \quad . \qquad (2.7)$$

The coefficients $L_i(x)$ are called the Lagrange coefficient polynomials.[1] Using these coefficients the unique polynomial $P(x)$ of degree $\leq n$ which passes through n+1 points and has the property that

$$P(x_i) = f(x_i) \qquad i = 0, 1, \ldots n$$

is given by

$$P(x) = L_0(x)\ f(x_0) + L_1(x)\ f(x_1) + \ldots + L_n(x)\ f(x_n) \qquad (2.8)$$

where the $L_i(x)$ are polynomials of degree n and $P(x)$ is of degree $\leq n$.

The interval [0, n] over which a polynomial can approximate a function is used to measure the length of the lag. For

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[1]Moursund and Duris (1967, p. 129).

example, if one wishes to see if the lag on income is six periods long, n is set equal to six. Since the precise length of the lag is not known in advance the lags are varied and the best is chosen.

In theory the choice of the degree of the polynomial is constrained by the fact that it must be less than or equal to n. The restriction placed on the polynomial by the programming package used here is slightly different. The polynomial must be less than n, but can be no greater than fourth degree nor smaller than third degree. This means that when constructing a third degree polynomial it is necessary to have four known points for the polynomial to pass through and five known points in the case of a fourth degree polynomial. These points are chosen from the interval [0, n]. Theoretically it makes no difference where the points are located in the interval. The intuitive explanation is that within the interval there can only be one polynomial which minimizes the sum of squares. However, the programming package requires that one of the points chosen must be set equal to n. The advantage of this method is that we are estimating a few points on the curve and polynomial interpolation will interpolate between them for the remaining points.

In order to obtain estimates of the elasticity coefficients and weights the interpolation distribution assumes that the $\alpha w(i)$ are values of a polynomial of degree $\leq n$.

Once n + 1 points on the curve are known- $\alpha_1 w(i_0) = b_0$, $\alpha_1 w(i_1) = b_1$ ,... $\alpha_1 w(i_n) = b_n$- all the $\alpha_1 w(i)$ can be calculated as linear combinations of the known values.

For example, let us assume that the $\alpha_1 (w_i)$ are values of a third degree polynomial and that the length of the lag is six periods. Four points must be chosen from the interval [0, 6). Assume that the following points are known:

$$\alpha_1 (w_0) = b_1$$

$$\alpha_1 (w_1) = b_2$$

$$\alpha_1 (w_2) = b_3$$

$$\alpha_1 (w_3) = b_4 = 0$$

The setting of $b_4$ equal to zero is a restriction of the programming package. In Equation 2.8 $\alpha_1 (w_i)$ can be substituted for P(x) and $b_k$ for f(x) to yield

$$\alpha_1 (w_i) = L_1 (x) b_1 + L_2 (x) b_2 + L_3 (x) b_3 + L_4 (x) b_4 \qquad (2.9)$$

This can be further simplified because of a restriction of the programming package that $b_4$ be set equal to zero. Therefore, we are left with

$$\alpha_1 w(i) = L_1 (x) b_1 + L_2 (x) b_2 + L_3 (x) b_3 \qquad (2.10)$$

Substituting Equation 2.10 into Equation 2.3 gives

$$
\begin{aligned}
M_t^d = \alpha_0 &+ L_1(0)b_1Y_t + L_2(0)b_2Y_t + L_3(0)b_3Y_t \\
&+ L_1(1)b_1Y_{t-1} + L_2(1)b_2Y_{t-1} + L_3(1)b_3Y_{t-1} \\
&\;\vdots \\
&+ L_1(6)b_1Y_{t-6} + L_2(6)b_2Y_{t-6} + L_3(6)b_3Y_{t-6} \\
&+ u_t
\end{aligned}
\tag{2.11}
$$

Equation 2.11 can be rearranged to give

$$
\begin{aligned}
M_t^d = \alpha_0 &+ b_1\ [L_1(0)Y_t + L_1(1)Y_{t-1} +\ldots+ L_1(6)Y_{t-6}] \\
&+ b_2\ [L_2(0)Y_t + L_2(1)Y_{t-1} +\ldots+ L_2(6)Y_{t-6}] \\
&+ b_3\ [L_3(0)Y_t + L_3(1)Y_{t-1} +\ldots+ L_3(6)Y_{t-6}] \\
&+ u_t
\end{aligned}
\tag{2.12}
$$

The terms contained within the brackets of Equation 2.12 can be calculated quite easily. We have seen that the $L_i(x)$ are constants that can be computed independently of the data. The $Y_{t-i}$ values represent given data. These bracketed terms are called the Almon variables, which we will designate as $A_{1t}$, $A_{2t}$, and $A_{3t}$. The following equation is fitted by the method of least squares:

$$
M_t^d = \alpha_0 + b_1A_{1t} + b_2A_{2t} + b_3A_{3t} + u_t \ . \tag{2.13}
$$

Since the Almon variables are known, the fitting of Equation 2.13 will provide us with estimates of the b's. Given the b's we can then proceed to derive the estimates of $\alpha_1$ and the weights.

In order to derive the estimates of $\alpha_1$ and the weights we can rewrite Equation 2.10 as

$$\alpha_1 w(i) = \sum_{k=1}^{3} L_i(x) b_k \tag{2.14}$$

The restriction that $\sum_{i=0}^{n} w(i) = 1$ means that

$$\alpha_1 \sum_{i=0}^{n} w(i) = \alpha_1$$

Now

$$\alpha_1 \sum_{i=0}^{n} w(i) = \sum_{i=0}^{n} \left[ \sum_{k=1}^{3} L_i(x) b_k \right]$$

$$= \sum_{k=1}^{3} \left[ \sum_{i=0}^{n} L_i(x) b_k \right]$$

$$= \sum_{k=1}^{3} b_k \left[ \sum_{i=0}^{n} L_i(x) \right] \tag{2.15}$$

So that an estimate of $\alpha_1$ is given by

$$\hat{\alpha}_1 = \hat{b}_1 \sum_{i=0}^{n} L_1(x) + \hat{b}_2 \sum_{i=0}^{n} L_2(x) + \hat{b}_3 \sum_{i=0}^{n} L_3(x)$$

We have the estimates of the b's, $\alpha_1$, and the values of the L(x)'s. Then, since

$$\hat{\alpha}_1 \; \hat{w}(i) = \sum_{k=1}^{3} L_i(x)\,[\hat{b}_k]$$

the particular values of w(x) are found by

$$\hat{w}(i) = \frac{\sum_{k=1}^{3} L_i(x)\,[\hat{b}_k]}{\hat{\alpha}_1}$$

The variance of $\hat{\alpha}_1$ can be computed as

$$\text{VAR}\;[\hat{\alpha}_1 \hat{w}(i)] = \sum_{k=1}^{3} \;[l_i(x)]^2 \;\text{VAR}(\hat{b}_k)$$

$$\text{VAR}\;[\hat{\alpha}_1] = \sum_{k=1}^{3} \left[\sum_{i=0}^{n} l_i(x)\right] \text{VAR}(\hat{b}_k)$$

And the variance of w(x) is found, based on Fieller's (1940) theorem, by

$$\text{VAR}[w(i)] = \frac{\text{VAR}[\alpha_1 w(i)] - 2w(i)\;\text{COV}[\alpha_1 w(i)\alpha_1] + w(i)^2 \text{VAR}(\alpha_1)}{\alpha_1^{\;2}}$$

In summary, the computational steps we have followed are:

1. select the degree of the polynomial and the length of the lag

2. from the interval [0, n] arbitrarily choose points equal to one plus the degree of the polynomial for the polynomial to pass through.
3. calculate the Lagrange Interpolation Coefficients $L_i(x)$
4. compute the Almon variables
5. apply least squares to estimate the $b_k$ from

$$M_t^d = \alpha_0 + b_1 A_{1t} + b_2 A_{2t} + b_3 A_{3t} + u_t$$

6. using the estimates of $b_k$ solve for the values of $\alpha_1$ and the w(i)'s.

This example has been based on only one explanatory variable of lag length n. However, as we shall see, the method can easily be extended to allow for a number of explanatory variables of differing lag lengths. The only restriction with more than one lagged variable, and it is a programming restriction, is that the polynomial must be of the same degree for all variables.

# CHAPTER III. THE MODEL

According to Starleaf (1970) the empirical estimation of a demand function for money requires the specification of at least three factors. They are:

1) a demand function for money
2) a supply function for money
3) a specified relationship between the demand for and the supply of money.

In this study the supply of money is assumed to be exogenously determined. In addition, the supply of money is assumed to equal the demand for money at all times.

These assumptions mean that we are dealing with a very simple money demand - supply model. Fortunately, for the purposes of this study, a simple model should be adequate. As mentioned in chapter one, studies using models containing simultaneous equations to take account of the endogeneity of the money supply produced estimates of elasticity coefficients similar to those estimated from single equation models. Some studies have also made allowances for a systematic inequality between the demand for and the supply of money. However, Starleaf (1970) has tested the proposition that there was a systematic lag for the years 1952 - 1966 and found no lag to exist for a period of time at least as short as one quarter. Therefore, we see that there are good reasons for keeping the model simple.

The general demand function, which is assumed to be linear in the logarithms, used in this study is

$$\ln M_t^d = B_0 + B_y \ln Y_t^* + B_R \ln R_{Rt}^* + B_r \ln R_{rt}^* + B_p \ln P_t + u_t \qquad (3.1)$$

where

$M_t^d$ = the nominal stock of money

$$\ln Y_t^* = \sum_{i=0}^{n_Y} (\ln Y_{t-i})\, W_Y(i)$$

$$\ln R_{Rt}^* = \sum_{i=0}^{n_R} (\ln R_{Rt-i})\, W_R(i)$$

$$\ln R_{rt}^* = \sum_{i=0}^{n_r} (\ln R_{rt-i})\, W_r(i)$$

$Y_{t-i}^*$ = GNP in period t-i

$R_{Rt-i}^*$ = rate of interest on 4 to 6 month commercial paper in period t-i

$R_{rt-i}^*$ = rate of interest on time deposits in period t-i

$P_t$ = implicit price deflator in period t

$B_0$ = intercept term

$B_y$ = the elasticity of money demand with respect to $Y_t^*$

$B_R$ = the elasticity of money demand with respect to $R_{Rt}^*$

$B_r$ = the elasticity of money demand with respect to $R_{rt}^*$

$B_p$ = the elasticity of money demand with respect to $p_t$

$u_t$ = the error term

The fact that $B_Y$ is an elasticity coefficient can be demonstrated in the following manner:

$$M_t^d = B_0 Y_t^{*B_Y} R_t^{*B_R} \qquad (3.2)$$

Although this equation is exponential in form, a logarithmic transformation will make it linear in the logs and it will correspond to Equation 3.1 except for the fact that there is only one interest rate included in the exponential form. Let $E_{MY}$ represent the income elasticity of money demand which is

$$E_{MY} = \frac{\partial M_t^d / M_t^d}{\partial Y_t^* / Y_t^*} = \frac{\partial M_t^d}{\partial Y_t^*} \cdot \frac{Y_t^*}{M_t^d}$$

From Equation 3.2

$$\frac{\partial M_t^d}{\partial Y_t^*} = B_Y (B_0 Y_t^{*B_Y - 1}) R_t^{*B_R}$$

Therefore,

$$E_{MY} = B_Y (B_0 Y_t^{*B_Y - 1}) R_t^{*B_R} \frac{Y_t^*}{B_0 Y_t^{*B_Y} R_t^{*B_R}}$$

$$E_{MY} = B_Y$$

The same procedure could be used to demonstrate that $B_R$, either the coefficient on commercial paper or on time deposits, is an elasticity coefficient. The procedure could also be used to demonstrate that the weights $W_Y(i)$, $W_R(i)$, and $W_n(i)$ are also elasticities.

As previously stated the purpose of this study is to apply the general model shown in 3.1 to monthly, quarterly, and annual data in order to estimate both the elasticity coefficients and the weight structures associated with these different frequencies of data. In so doing it is hoped that a determination can be made as to whether the frequency of data used exerts any influence over the estimates of the elasticities and the weights. And, if any influence is present, to note its direction.

All reported tests on quarterly data were carried out using a third degree polynomial. Previous tests by Dickson (1969) on quarterly data have shown that considering the length of the lag, goodness of fit, and computing costs, the third degree polynomial was superior to polynomials of any higher degree.

The tests on annual data were also carried out with a third degree polynomial. Results from quarterly data indicated a lag on both income and interest rates of from four to five quarters. If the length of the lag is fairly constant, irrespective of the frequency of data used, then the lag lengths using annual data should be quite short. The shortest lag that can be used is one plus the degree of the polynomial. It would have been desirable to use a second degree polynomial so that a three year, rather than a four year, lag could have been imposed on the annual data. However, the programming capacities did not allow for it. A third degree polynomial was the lowest that could be used.

Both third and fourth degree polynomials were tried in calculating the weight structure for monthly data. And, although the weights generated using monthly data were disappointing no matter what degree of polynomial was used, the fourth degree was more successful in yielding nonnegative weights. Therefore, reported monthly regressions use a fourth degree polynomial.

Tests of significance on both the coefficients and weights were t tests carried out at the five per cent level of significance. Because of the shortness of most of the lags, the lags were extended as long as they were positive, whether they were statistically significant or not.

All of the data, with the exception of the time deposit interest rate, was compiled from the data bank of Data Resources Incorporated (DRI). This is a service which provides various types of data and statistical packages to users through a teletype tied into a main computer. The data on nominal money stock, seasonally adjusted, the rate on three month Treasury Bills, and the rate on four to six month commercial paper were compiled from DRI based on the statistics collected by the Federal Reserve System and published in the

Federal Reserve Bulletins. The nominal Gross National Product and implicit price deflator series were originally compiled by the Bureau of Economic Analysis for the Department of Commerce and can be found in the Survey of Current Business. Both are seasonally adjusted. The time deposit interest rate was compiled from the 1957, 1965, and the 1972 issues of the Annual Report of the Federal Deposit Insurance Corporation.

Short-term rates of interest were used rather than long-term rates because it was felt that they provide a good measure of the opportunity cost of holding money. Also, they are quite free from risk and are more sensitive to economic changes than are long-term rates. In including a second yield, the rate on time deposits, we are taking the position that time deposits are good substitutes for $M_1$ in people's portfolios. And since there is a legal ceiling on maximum time deposit rates that was held below the market determined rate during the test period, a market determined interest rate would not be an acceptable proxy.

Data on prices was gathered in order to test for money illusion in the demand for money and to deflate the nominal money and income series. Current rather than lagged prices were used because lagged prices failed to generate a significant lag strucutre for more than two quarters in the past.

Data Resources Incorporated gives the money stock, the rate on three month treasury bills, and the rate on four to six month commercial paper only as a monthly series. GNP and the implicit price deflator are given on an annual and quarterly basis, but not monthly. However, the DRI system also provides a technique for converting a series from a high (monthly) frequency to a lower (quarterly) series or vice versa. The movement from a high to a low frequency series is done by averaging. For example, the conversion of a monthly to a quarterly series is accomplished by summing three months and dividing by three in order to obtain a quarterly estimate. In the case of distributing a low series as a high series the new series to be created will have the same average as the old series, but the movements of a third series.[1] For example, the first quarter GNP for 1968 is $834.00. The first three monthly observations for 1968 are $824.077, $833.749, and $844.174. The average of these three observations is equal to the quarterly observation. The specific values for the monthly observations are found by taking a monthly series and multiplying it by a conversion factor to arrive at the monthly estimates. The monthly series used to distribute GNP was personal income. In the example cited here, multiplying personal income for the first three months of 1968

[1]Hall (1971, p. E40).

($656.1, $663.8, and $672.1) by approximately 1.256 will yield the monthly estimates of GNP. Therefore, all data taken from the DRI was ultimately available in all three frequencies.

The time deposit interest rate represented the only data not collected from the DRI. It was available only on an annual basis. The quarterly and monthly rates were found by straight line interpolation.

Quarterly and annual data were collected for the period from 1949 to 1970. The actual period used in the quarterly and annual regressions does not begin until the first quarter of 1952 and 1954 respectively. The early years were used in the creation of the Almon variables. Monthly data was collected for the period 1950 to 1970. The regressions start with the first month of 1952.

## Serial Correlation

It has already been mentioned that one of the advantages of using the Lagrange Interpolation Polynomial Technique was that it did not require any beforehand assumptions about the particular shape of the weight structure. Using this technique the weight structures generated could be j-shaped, u-shaped, an inverted u, continuously declining, or a number of other shapes. Theoretical considerations of the demand for money, however, indicate that certain lag shapes are unacceptable. Negative weights for any variable are generally considered unrealistic. Furthermore, it seems unlikely that the weights on any of the independent variables should be u or v-shaped or have multiple peaks. For example, there is no reason to expect the effect of income on the demand for money to be great in the early quarters, fall off in the later quarters, and finally to rise again as the end of the lag is reached. Rather, we would expect the effect to continuously decline, or to rise, reach a maximum, and then fall off.

The initial weights generated in this study were disappointing. Negative weights were found for both the income and interest rate variables. In those instances where the weights were all positive, the structure was often times u-shaped. Besides a priori expectations about the shape of the lags, it was also noted that the Durbin-Watson statistic was quite low (0.3 to 0.5). The low Durbin-Watson statistic is indicative of serial correlation of the residuals.

Fuller and Martin (1961) in discussing the effects of serial correlation on the estimation of distributed lag models give three reasons for expecting serial or autocorrelation in the errors:[1]

1) given autocorrelated series the choice of an incorrect functional form will result in autocorrelated errors;

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[1]Fuller and Martin (1961, p. 72).

2) errors arising from the omission of economic and non-economic variables will tend to be autocorrelated, as the omitted variables are generally autocorrelated;

3) errors of measurement present in the data are often autocorrelated.

If the error terms are serially correlated, the technique of ordinary least squares will not yield the best linear unbiased estimates. According to the Gauss-Markov Theorem, ordinary least squares will provide the minimum variance unbiased estimate only when the error terms are serially independent and have the same variance for all observations.[1] A less restrictive error model must be used. Rather than assume that the errors ($u_t$) are uncorrelated, we will assume that the errors follow a first order autoregressive scheme

$$u_t = \rho\, u_{t-1} + v_t \tag{3.3}$$

where

$E(V_i V_j) = 0 \qquad i \neq j$

$E(V_i V_j) = \text{constant} \qquad i=j \text{ for all } j$

In order to examine the procedure used in dealing with serial correlation in the errors consider the equation

$$M_t^d = \alpha_0 + \alpha_1\, Y_t + u_t \tag{3.4}$$

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[1]Rao and Miller (1971, p. 69).

where $Y_t$ is current income and the $u_t$'s are serially correlated. Equation 3.4 is identical to Equation 2.1 except that we need not concern ourselves here with lagged values of income. A transformation must be performed on the serially correlated error term in order to generate a new error term which is serially independent. From Equation 3.3 we get

$$u_t^* = u_t - \rho\, u_{t-1} \tag{3.5}$$

where $u_t^* = v_t$.

The same transformation must be performed on the dependent and independent variables in 3.4 so that we are left with

$$M_t^d - \rho\, M_{t-1}^d = \alpha_0(1-\rho) + \alpha_1(Y_t-\rho Y_{t-1}) + (u_t-\rho u_{t-1}) \tag{3.6}$$

Following the notation of 3.5 we can let $M_t^{d*}$, $\alpha_0^*$, and $\alpha_1\, Y_t^*$ equal $M_t^d-\rho M_{t-1}$, $\alpha_0(1-\rho)$, and $\alpha_1(Y_t-\rho Y_{t-1})$ and we are left with

$$M_t^{d*} = \alpha_0^* + \alpha_1 Y_t^* + u_t^* \tag{3.7}$$

In this equation the error terms are serially independent (non-autocorrelated), and of constant variance, that is:

$$E(u_t^* u_{t-1}^*) = 0$$

$$E(u_t^{*2}) = \sigma^2$$

This method of estimation of the parameters from transformed data is called generalized least squares. As has been seen, once the parameter is known, the transformation is made and ordinary least squares estimation will provide the minimum variance unbiased estimates of the coefficients. However, this holds true only when the true value of $\rho$, the autocorrelation coefficient, is known. In practice this value is not known, but must be estimated. $\hat{\rho}_1$, an estimate of $\rho$, is found from the residuals of ordinary least squares estimation as

$$\hat{\rho}_1 = \frac{\Sigma\ u_t u_{t-1}}{\Sigma\ u_{t-1}^2} \tag{3.8}$$

The method of generalized least squares was used in this study to correct for serial correlation. Generally it was successful in raising the Durbin-Watson statistic from a range of 0.3 to 0.5 to approximately 1.0. This eliminated the negative weights and helped to smooth out the weight structure. However, some of the lags still tended to be u-shaped. Since the Durbin-Watson was not high enough to accept the hypothesis of zero autocorrelation at the five per cent level of significance, a method was used which combines finding the proper $\rho$ and the correct autoregressive order. That method was to transform the transformed variables.

Following the last example, if the Durbin-Watson was still not up to acceptable levels a second estimate of $\rho$, call it $\hat{\rho}_2$, was found by the regression

$$u_t^* = + \rho_2 u_{t-1}^* + V_t' \tag{3.9}$$

The $\hat{\rho}_2$ is used to transform Equation 3.7 which becomes

$$M_t^{d*} - \hat{\rho}_2 M_{t-1}^{d*} = \alpha_0^*(1-\hat{\rho}_2) + \alpha_1(Y_t^* - \hat{\rho}_2 Y_{t-1}) + (u_t^* - \hat{\rho}_2 u_{t-1}^*) \tag{3.10}$$

Again, following the notation pattern of 3.5 this can be rewritten as

$$M_t^{d**} = \alpha_0^{**} + \alpha_1 Y_t^{**} + u_t^{**} \tag{3.11}$$

In order to examine Equation 3.11 in terms of the original data we can concentrate on any one variable. For example:

$$\begin{aligned} M_t^{d**} &= M_t^{d*} - \hat{\rho}_2 M_{t-1}^{d*} \\ &= M_t^d - \hat{\rho}_1 M_{t-1}^d - \hat{\rho}_2 (M_{t-1}^d - \hat{\rho}_1 M_{t-2}^d) \\ &= M_t^d - \hat{\rho}_1 M_{t-1}^d - \hat{\rho}_2 M_{t-1}^d + \hat{\rho}_1 \hat{\rho}_2 M_{t-2}^d \\ &= M_t^d - (\hat{\rho}_1 + \hat{\rho}_2)\, M_{t-1}^d + \hat{\rho}_1 \hat{\rho}_2 M_{t-2}^d \end{aligned} \tag{3.12}$$

This transformation of the transformed data indicates that we are allowing for a kind of second-order serial correlation.

The Durbin-Watson is again checked to see if it is up to acceptable levels. In general, it took from two to three iterations of the data used in this study to raise the Durbin-Watson statistic to a level high enough to accept the hypothesis of zero autocorrelation at the five per cent level of significance. Once this was done the lag structures conformed to the a priori expectations of either continuously declining, or rising, reaching a maximum, and then falling off.

# CHAPTER IV. RESULTS AND DISCUSSION

This chapter is a discussion of the findings of this study. The Appendix contains the estimated money demand equations which are discussed here. The tables in this chapter are comparisons of some of the results from the Appendix. The quarterly, annual, and monthly findings will be presented in turn.

## Quarterly

Table 1 compares the elasticity coefficients and weights for the estimated money demand equations where the dependent variable is first, the nominal, and second the real stock of money narrowly defined. In both cases all of the coefficients are significant and of the right sign. The differences between the coefficients are quite small. Using the nominal as opposed to the real stock of money as the dependent variable gave a slightly higher income and time deposit rate elasticity (.005 and .039 higher, respectively), but resulted in a lower interest elasticity (by .041) on commercial paper.

The weight structures for the independent variables were also quite similar. Both sets of weights associated with income were four quarters in length, shaped like an inverted-u, and peaked in the second quarter. The weights on commercial paper were four quarters in length and declined continuously.

Table 1. Comparisons of the elasticities and weight structures for the nominal and real stock of money

| Dependent variables | $M_1$ | | | | $\frac{M_1}{P}$ | | |
|---|---|---|---|---|---|---|---|
| Independent variables | GNP | RCMP | RTD | P | $\frac{GNP}{P}$ | RCMP | RTD |
| Elasticity | .848 | -.0714 | -.263 | .325 | .843 | -.0755 | -.224 |
| t-statistic | (164.75) | (-6.765) | (-6.972) | (1.948) | (419.74) | (-6.823) | (-14.546) |
| Weights | | | | | | | |
| w(0) | 0.210<br>(3.949) | 0.314<br>(6.623) | 0.109<br>(0.613) | | 0.225<br>(3.818) | 0.320<br>(6.215) | 0.057<br>(0.184) |
| w(1) | 0.272<br>(6.436) | 0.273<br>(7.547) | 0.217<br>(1.312) | | 0.270<br>(5.364) | 0.271<br>(7.088) | 0.275<br>(0.850) |
| w(2) | 0.295<br>(11.08) | 0.243<br>(10.258) | 0.345<br>(3.876) | | 0.287<br>(9.743) | 0.240<br>(9.322) | 0.372<br>(2.403) |
| w(3) | 0.223<br>(5.033) | 0.170<br>(4.626) | 0.329<br>(2.137) | | 0.217<br>(4.221) | 0.169<br>(4.373) | 0.297<br>(1.188) |
| w(4) | 0.000 | 0.000 | 0.000 | | 0.000 | 0.000 | 0.000 |

In the case of the time deposit interest rate, both weights were again four quarters in length, shaped like an inverted-u, and peaked in the second quarter. We can see that deflating both money and income by the price level had no appreciable effect on either the coefficients or the structure of the weights.

Since the reasons for including a time deposit interest rate in a money demand equation have already been given, it is not necessary to go into them again. And a glance at Table 1 would seem to indicate that empirically the rate on time deposits is important in formulating demand for money equations. The coefficient is the right sign, it is significant, and the lag structure seems reasonable. Table 2 compares the regression results from a money demand equation with the rate on time deposits included and excluded. The estimated coefficients of income, the rate on commercial paper, and price do not change appreciably when the rate on time deposits is dropped as an independent variable. There is also no change in the lag structure associated with the income variable. In both regressions presented in Table 2 the weights go back four quarters in length, are shaped like an inverted-u, and reach their maximum in the second quarter. However, there is a change in the lag structure associated with the rate on commercial paper. Where the time deposit rate has been dropped the lag associated with commercial paper

Table 2. Comparison of regression results with an omitted variable

| Dependent variable | $M_1$ | | | | $M_1$ | | | |
|---|---|---|---|---|---|---|---|---|
| Independent variables | GNP | RCMP | RTD | P | GNP | RCMP | P | Trend |
| Elasticity | 0.848 | -.0714 | -.263 | 0.325 | 0.844 | -0.0711 | 0.217 | |
| t-statistic | (164.75) | (-6.765) | (-6.972) | (1.948) | (155.51) | (-5.207) | (1.28) | (-6.54) |
| Weights | | | | | | | | |
| w(0) | 0.210 | 0.314 | 0.109 | | 0.230 | 0.294 | | |
| | (3.949) | (6.623) | (0.613) | | (3.928) | (4.882) | | |
| w(1) | 0.272 | 0.273 | 0.217 | | 0.284 | 0.305 | | |
| | (6.436) | (7.547) | (1.312) | | (6.371) | (8.469) | | |
| w(2) | 0.295 | 0.243 | 0.345 | | 0.285 | 0.235 | | |
| | (11.08) | (10.258) | (3.876) | | (9.749) | (8.856) | | |
| w(3) | 0.223 | 0.170 | 0.329 | | 0.201 | 0.130 | | |
| | (5.033) | (4.626) | (2.137) | | (4.010) | (3.908) | | |
| w(4) | 0.000 | 0.000 | 0.000 | | 0.000 | 0.036 | | |
| | | | | | | (0.891) | | |
| w(5) | | | | | | 0.000 | | |

is five quarters long rather than four, and the shape of the lag has changed from continuously declining to an inverted-u.

Whether this change in the lag structure was due to the dropping of the time deposit rate is difficult to say. A comparison of the quarterly tables in the Appendix shows that in regressions where the rate on time deposits was omitted the market rate of interest displayed both four and five quarter lags that were either continuously declining or u-shaped. Other factors such as the inclusion of trend, the particular market rate of interest chosen, and the length of the time period examined could also have influenced the lag.

Table 3 compares the weight structures generated using two different short-term rates of interest, the four to six month rate on commercial paper and the three month rate on treasury bills. In both cases the weights on income are shaped like an inverted-u and reach their maximum in the second quarter. Both interest rates are also shaped like an inverted-u and, in this instance, reach their maximum in the first quarter. It appears that the choice of a proxy for the market rate of interest does not affect the shapes of the lags. In both cases the shapes of the lags were identical.

In these regressions where income was entered as a lagged variable the elasticity coefficient was always significant and had a value between .842 and .877. The lag was four quarters long, shaped like an inverted-u, and in all but two cases

Table 3. Comparison of regression results with two different rates of interest

| Dependent variables | $M_1$ | | | | $M_1$ | | |
|---|---|---|---|---|---|---|---|
| Independent variables | GNP | RCMP | P | Trend | GNP | R3TB | P |
| Elasticity | .844 | -.0711 | .217 | -.0057 | 0.876 | -.261 | 0.174 |
| t-statistic | (155.51) | (-5.207) | (1.28) | (-6.54) | (78.008) | (-4.851) | (0.924) |
| Weights | | | | | | | |
| w(0) | 0.230<br>(3.928) | 0.294<br>(4.882) | | | 0.242<br>(3.756) | 0.195<br>(3.402) | |
| w(1) | 0.284<br>(6.371) | 0.305<br>(8.469) | | | 0.271<br>(5.872) | 0.266<br>(6.911) | |
| w(2) | 0.285<br>(9.749) | 0.235<br>(8.856) | | | 0.279<br>(8.671) | 0.255<br>(9.149) | |
| w(3) | 0.201<br>(4.010) | 0.130<br>(3.908) | | | 0.208<br>(3.873) | 0.189<br>(5.922) | |
| w(4) | 0.000 | 0.036<br>(0.891) | | | 0.000 | 0.095<br>(2.282) | |
| w(5) | | 0.000 | | | | | |

peaked in the second quarter. Deflating, inclusion and exclusion of variables, and changing of the interest rate did nothing to substantially affect the elasticity coefficient, the length of the lag, or the structure of the lag.

The lagged treasury bill rate was used in estimating three of the quarterly money demand equations. The coefficients associated with the treasury bill rate were -.261, -.265, and -.279, and all were significant. In this case all of the lags were shaped like an inverted-u, with one peaking in the first quarter and the other two in the second. Two of the lags were five periods long and one was four.

Significant coefficients that ranged from -.0711 to -.0830 were found in all cases for the four to six month rate on commercial paper. In those instances where the lag was four quarters in length the weights were continuously declining. In the two cases where the lag was five quarters in length the shape switched to an inverted-u, peaking in the first quarter.

In general, the elasticity coefficients for the interest rate variables conformed to those reported in other studies. In most other studies the coefficients have varied from approximately -.10 to -1.0 depending on the definition of money used, the term to maturity of the interest rate used, and the frequency of the data. Using short rather than long-term rates should result in a lower elasticity because variations in short rates tend to be greater than variations in long rates. In addition, the use of $M_1$ rather than $M_2$ will tend to raise the estimated elasticity because changing interest rates will cause people to shift some of their money balances between $M_1$ and time deposits.

One variable that was somewhat disappointing on two counts was the price variable. First, price changes are not always readily perceived. It should take time for people to recognize these changes and adjust their money holdings accordingly. Therefore, it would seem reasonable that price should enter the money demand equation as a lagged variable. Second, in theoretical specifications the demand for money balances is usually postulated as being homogenous of degree one in prices. This means that empirical studies estimating demand for money equations should find that the elasticity coefficient associated with price should be equal to one. And empirical studies by such people as Meltzer (1963) and Dickson and Starleaf (1972) found this to be the case. In this study, however, the coefficient did not indicate an absence of money illusion.

The problem can be seen by examining a simple money demand model which constrains the price elasticity to unity. If interest rates and the error term are ignored for a moment the model may be written as

$$\frac{M_t^d}{P_t} = B_0 \left(\frac{Y_t^*}{P_t}\right)^{B_1} \tag{4.1}$$

then, taking the natural logarithms,

$$\ln M_t^d - \ln P_t = B_0 + B_1 \ln Y_t^* - B_1 \ln P_t.$$

By rearranging terms

$$\ln M_t^d = B_0 + B_1 \ln Y_t^* + (1-B_1) \ln P_t \tag{4.2}$$

the elasticity coefficient associated with the price variable is equal to $1-B_1$. Table 3 presents two estimates of the price coefficient. In one case the income elasticity is 0.876 so that $1-B_1$ yields 0.124. The estimated value for price was actually 0.174. In the second case the estimated income elasticity is 0.844 so that $1-B_1$ yields 0.156. The estimated value was actually 0.217. Estimates of price elasticity in Table 3 tend to support the hypothesis of no money illusion in the demand for money. However, a glance at the tables in the Appendix will show that the other estimates of price elasticity do not tend to support that hypothesis. For example, Table 11 contains the largest estimate of the price coefficient. The income elasticity is 0.851 and $1-B_1$ is 0.149. The estimated value was 0.491. This discrepancy is why estimates of the price elasticity coefficients were disappointing.

Outside of the fact that the implicit price deflator may not be an accurate reflection of prices faced in the economy, the only explanation for the results generated by the price variable is multicollinearity. Often times estimates of income and price elasticities of demand from time series data are difficult to interpret because the price and income series move together. When multicollinearity is present it may cause large standard errors for the coefficients. The result being that the coefficients are not significantly different from zero. It is also possible that the estimated coefficients become quite sensitive to the data set. Running a given regression for a longer or shorter period can cause large changes in the coefficients. This makes it difficult to get an accurate estimate of the coefficients associated with price and income. However, the evidence presented here does not indicate that multicollinearity is the problem. Most of the coefficients are significantly different from zero. They do not change greatly as the time period examined is increased. We are left with no good explanation for the performance of the price variable.

Some of the regressions include an independent variable introduced to correct for trend. When the variables of a regression move in the same direction because of general economic activity, as they do in money demand formulations, the relations indicated by regression results may be spurious.

The explicit introduction of time is designed to abstract from this influence. Regressions were run with and without the trend element. Since there was no difference in the magnitudes of the coefficients or the length or shape of the lags the trend variable was dropped.

## Annual

If the length of the lags on the independent variables are consistent, no matter what the frequency of the data used, we would expect that attempts to estimate lags using annual data and a third degree polynomial would meet with little success. This is because the shortest lag that can be generated with a third degree polynomial is four periods, in this case four years. Quarterly results indicate, however, that the lag is between four and five quarters for all variables. This is not long enough to generate much of a lag using annual data.

The attempts to estimate lags using annual data were not very successful. Table 18 of the Appendix shows the only reasonable lag that was generated. The income variable was the only one that yielded nonnegative weights, where the weight structure was shaped like an inverted-u with most of the impact (.505) coming one year in the past. The weight in the second year (.345) is still large and significant. By the third year the weight is small (.040) and insignificant.

Regressions using unlagged variables were run on the annual data in order to get elasticity coefficients to compare with those from quarterly data. As can be seen in Table 4, the coefficients derived from the annual data were quite similar to those found using quarterly data. The frequency of data used, at least quarterly versus annually, did not have any appreciable effect on the estimates of the elasticity coefficients.

## Monthly

Estimates of money demand equations using monthly data were obtained in the same manner as the quarterly and annual estimates. If the quarterly results concerning the length of the lags was representative of lag length in general, we would expect to find approximately a twelve period lag on the income and assorted interest rate variables. However, the results using monthly data were quite disappointing. Tables 22 and 23 in the Appendix show that the only variable that responded with a reasonable lag structure was income. All attempts at finding lag structures for the interest rate variables resulted in negative weights.

The weights associated with the income variable are nine periods in length and decline continuously. In the previous quarterly and annual regressions where income was a lagged variable the weights were shaped like an inverted-u rather

Table 4. Comparisons of regression coefficients computed from quarterly and annual data

| Frequency of data | Quarterly (1952-1966) | | | Annually (1952-1966) | | |
|---|---|---|---|---|---|---|
| Dependent variables | $\frac{M_1}{P}$ | | | $\frac{M_1}{P}$ | | |
| Independent variables | GNP | RCMP | RTD | GNP | RCMP | RTD |
| Elasticity | 0.843 | -0.0755 | -.224 | 0.841 | -.0774 | -0.242 |
| t-statistic | (419.74) | (-6.823) | (-14.546) | (90.628) | (-3.810) | (-10.373) |
| Independent variable | GNP | RCMP | | GNP | RCMP | |
| Elasticity | 0.842 | -.077 | | 0.851 | -0.103 | |
| t-statistic | (423.02) | (-7.430) | | (107.693) | (-5.557) | |

than declining continuously. The difference in lag structures associated with income cannot be attributed to the fact that a fourth rather than a third degree polynomial was used in the monthly regressions. Previous tests by Dickson (1969) demonstrated that the basic shape of the lag function did not change when the degree of the polynomial was altered.

A possible explanation could be that an effect of changing the frequency of the data is to alter the shape of the lag. Changing the frequency of the data, it could also be argued, resulted in the length of the lag being shorter than that which would have been predicted from quarterly or annual results. However, this explanation regarding the reason for the length of the lag cannot be accepted. The results from the monthly data are too meager to support such a conclusion. And although such an explanation cannot be ruled out, it cannot be accepted either.

The source of the problem may lie with the quality of the data. As explained in Chapter III, some of the monthly data was found by interpolation from annual and quarterly data. This interpolation process is not likely to yield the same parameters that would have been found had the data actually been gathered on a monthly basis. There is also a question as to the advisability of assuming that money demand is equal to money stock for a period of time as short as one month. As already mentioned previous tests have indicated that there is equality of money demand and money stock for a period of time as short as one quarter. However, there is no such evidence for one month. So it would seem that the problem of obtaining reasonable lag structures for monthly data may lie with the quality of the data or with the specification of the money demand model.

A glance at Table 5 shows that the income elasticity of money demand was quite consistent regardless of the type of data used. The coefficient attached to the rate on time deposits also displayed the same consistency irrespective of the frequency of the data. The elasticity coefficient associated with the rate on commercial paper for monthly data appears to be somewhat lower than those coefficients generated with lower frequencies of data. However, if the monthly results are compared with the quarterly results for the current, rather than the lagged rate on commercial paper as in Tables 16 and 17 of the Appendix, it becomes evident that the interest elasticity associated with commercial paper is quite similar irrespective of the frequency of data used.

## Conclusion

In conclusion, it appears that the frequency of data used has had no appreciable effect on the magnitudes of any of the elasticity coefficients. The coefficients have also been within the range of estimates found by others in their empirical

Table 5. Comparisons of regression coefficients computed from quarterly, annual, and monthly data

| Frequency of data | Quarterly (1952-1966) | | | Annually (1952-1966) | | | Monthly (1952-1966) | | |
|---|---|---|---|---|---|---|---|---|---|
| Dependent variables | Real $M_1$ | | | Real $M_1$ | | | Real $M_1$ | | |
| Independent variables | GNP | RCMP | RTD | GNP | RCMP | RTD | GNP | RCMP | RTD |
| Elasticity | 0.843 | -0.0755 | -.224 | 0.841 | -0.0774 | -0.242 | 0.836 | -.0180 | -0.268 |
| t-statistic | (419.74) | (-6.823) | (-14.546) | (90.628) | (-3.810) | (-10.373) | (498.77) | (-3.796) | (-33.436) |
| Independent variables | GNP | RCMP | | GNP | RCMP | | GNP | RCMP | |
| Elasticity | 0.842 | -.077 | | 0.851 | -0.103 | | 0.834 | -.0156 | |
| t-statistic | (423.02) | (-7.430) | | (107.693) | (-5.557) | | (201.877) | (-3.074) | |

studies of money demand equations.

Nothing conclusive can be said about the degree of influence, or lack of influence, that changing the frequency of the data had on the length or shape of the lag structures. The results from the annual and monthly data were too sparse.

# CHAPTER V. SUMMARY

This study estimated money demand equations using the Lagrange Interpolating Polynomial Technique, a technique which has the advantage of allowing the data to determine the shape of the lag structure associated with the independent variables. The purpose was to estimate and compare the elasticity coefficients and weight structures for different frequencies of data. It was hoped that such comparisons would provide an indication as to any bias or lack of bias in the coefficients and weight structures resulting from changing the frequency of the data.

Money demand equations containing income, interest rates, and prices were estimated using generalized least squares. It was found that the frequency of data used had no appreciable effect on the size of any of the elasticity coefficients. Because of the paucity of results concerning the lag structures for monthly and annual regressions, no conclusions about the effects of the frequency of data on the length or shape of the lag structures could be reached.

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# APPENDIX. TABLES OF RESULTS

Table 6. Quarterly regression (1952-1966) of money on lagged income, lagged rate on commercial paper, lagged rate on time deposits, price and trend

Lag polynomial degree: Number of iterations: 3

<u>Before iterations</u>: R-Bar Squared: 0.995 Durbin-Watson Statistic: 0.4968

<u>After iterations</u>: R-Bar Squared: 0.999 Durbin-Watson Statistic: 2.0160

Elasticities: $B_Y = 0.850$, (163.64) $B_{RCMP} = -.0711$, (-.0732) $B_{RTD} = -.163$, (-3.163) $B_p = 0.435$ (2.544)

$B_t = -.003$, (-2.665) $B''_o = .0027$; $\hat{p}_1 = 0.735$, (8.058) $\hat{p}_2 = 0.441$, (3.264) $\hat{p}_3 = 0.182$ (1.371)

| Lagged independent variables | Quarters in the past 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| GNP | | | | | |
| w(i) | <u>0.224</u> | <u>0.274</u> | <u>0.288</u> | <u>0.214</u> | <u>0.000</u> |
| StE(i) | 0.054 | 0.044 | 0.027 | 0.045 | 0.000 |
| t | 4.121 | 6.282 | 10.591 | 4.706 | 0.000 |
| 4-6 mo. rate | | | | | |
| w(i) | <u>0.313</u> | <u>0.273</u> | <u>0.243</u> | <u>0.171</u> | <u>0.000</u> |
| StE(i) | 0.049 | 0.038 | 0.025 | 0.038 | 0.000 |
| t | 6.360 | 7.209 | 9.878 | 4.496 | 0.000 |
| Time deposit rate | | | | | |
| w(i) | <u>0.032</u> | <u>0.267</u> | <u>0.284</u> | <u>0.316</u> | <u>0.000</u> |
| StE(i) | 0.294 | 0.274 | 0.147 | 0.242 | 0.000 |
| t | 0.109 | 0.977 | 2.608 | 1.307 | 0.000 |

Table 7. Quarterly regression (1952-1966) of the money supply (narrow) on lagged income, lagged rate on commercial paper, lagged rate on time deposits, and price

Lag polynomial degree: 3 Number of iterations: 3

Before iterations: R-Bar Squared: 0.995 Durbin-Watson Statistic: 0.4970

After iterations: R-Bar Squared: 0.999 Durbin-Watson Statistic: 1.9977

Elasticities: $B_y = 0.848$, (164.75) $B_{RCMP} = -.0714$, (-6.765) $B_{RTD} = -.263$, (-6.972) $B_p = 0.325$, (1.948)

$B_0'' = .0016$ ; $p_1 = 0.732$, (8.176) $p_2 = 0.699$, (6.379) $p_3 = .069$ (0.512)

| Lagged independent variables | Quarters in the past 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| GNP | | | | | |
| w(i) | 0.210 | 0.272 | 0.295 | 0.223 | 0.000 |
| StE(i) | 0.053 | 0.042 | 0.027 | 0.044 | 0.000 |
| t | 3.949 | 6.436 | 11.080 | 5.033 | 0.000 |
| 4-6 mo. rate | | | | | |
| w(i) | 0.314 | 0.273 | 0.243 | 0.170 | 0.000 |
| StE(i) | 0.047 | 0.036 | 0.024 | 0.037 | 0.000 |
| t | 6.623 | 7.547 | 10.258 | 4.626 | 0.000 |
| Time deposit rate | | | | | |
| w(i) | 0.109 | 0.217 | 0.345 | 0.329 | 0.000 |
| StE(i) | 0.178 | 0.165 | 0.089 | 0.154 | 0.000 |
| t | 0.613 | 1.312 | 3.876 | 2.137 | 0.000 |

Table 8. Quarterly regression (1952-1966) of money on lagged income, lagged rate on treasury bills, price and trend

Lag polynomial degree: 3 — Number of iterations: 3

Before iterations: R-Bar Squared: 0.993 — Durbin-Watson Statistic: 0.3091

After iterations: R-Bar Squared: 0.999 — Durbin-Watson Statistic: 1.9150

Elasticities: $B_y = 0.882$, (78.232), $B_{R3TB} = -.265$, (-4.808), $B_p = 0.410$, (2.394), $B_t = -.0049$, (-4.252)

$B''_o = .0006$ ; $\hat{p}_1 = 0.23$[illegible], (13.414), $\hat{p}_2 = 0.611$, (6.331), $\hat{p}_3 = 0.259$ (2.139)

| Lagged independent variables | Quarters in the past: 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| GNP | | | | | | |
| w(i) | 0.226 | 0.266 | 0.287 | 0.221 | 0.000 | |
| StE(i) | 0.061 | 0.045 | 0.030 | 0.051 | 0.000 | |
| t | 3.704 | 5.948 | 9.435 | 4.300 | 0.000 | |
| RTB | | | | | | |
| w(i) | 0.173 | 0.257 | 0.259 | 0.203 | 0.109 | 0.000 |
| StE(i) | 0.045 | 0.034 | 0.023 | 0.027 | 0.036 | 0.000 |
| t | 3.800 | 7.598 | 11.497 | 7.540 | 2.988 | 0.000 |

Table 9. Quarterly regression (1952-1966) regression of money on lagged income, lagged rate on treasury bills, price and trend

Lag polynomial degree: 3 — Number of iterations: 3

Before iterations: R-Bar Squared: 0.991 — Durbin-Watson Statistic: 0.4031

After iterations: R-Bar Squared: 0.999 — Durbin-Watson Statistic: 1.9531

Elasticities: $B_y = 0.876$, (78.008) $B_{R3TB} = -.261$, (-4.851) $B_p = 0.174$, (0.924) $B_t = -.0045$, (-4.363)

$B''_o = .0023$; $\hat{p}_1 = 0.781$, (9.901) $\hat{p}_2 = 0.601$, (5.316) $\hat{p}_3 = 0.209$ (1.586)

| Lagged independent variables | Quarters in the past 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| GNP | | | | | | |
| w(i) | 0.242 | 0.271 | 0.279 | 0.208 | 0.000 | |
| StE(i) | 0.064 | 0.046 | 0.032 | 0.054 | 0.000 | |
| t | 3.756 | 5.872 | 8.671 | 3.873 | 0.000 | |
| RTB | | | | | | |
| w(i) | 0.195 | 0.266 | 0.255 | 0.189 | 0.095 | 0.000 |
| StE(i) | 0.057 | 0.038 | 0.028 | 0.032 | 0.042 | 0.000 |
| t | 3.402 | 6.911 | 9.149 | 5.922 | 2.282 | 0.000 |

Table 10. Quarterly regression (1952-1966) of money on lagged income, lagged rate on commercial paper, price and trend

| | |
|---|---|
| Lag polynomial degree: 3 | Number of iterations: 3 |
| Before iterations: R-Bar Squared: 0.993 | Durbin-Watson Statistic: 0.4581 |
| After iterations: R-Bar Squared: 0.999 | Durbin-Watson Statistic: 1.9653 |

Elasticities: $B_Y = 0.844$, (155.51) $B_{RCMP} = -.0711$, (-5.207) $B''_o = .0019$, $B_p = .217$, (1.28) $B_t = -.0057$ (-6.54)

$\hat{p}_1 = 0.750$, (8.905) $\hat{p}_2 = 0.529$, (4.476) $\hat{p}_3 = 0.229$ (1.747)

| Lagged independent variables | Quarters in the past 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| GNP | | | | | | |
| w(i) | 0.230 | 0.284 | 0.285 | 0.201 | 0.000 | |
| StE(i) | 0.058 | 0.045 | 0.029 | 0.050 | 0.000 | |
| t | 3.928 | 6.371 | 9.749 | 4.010 | 0.000 | |
| 4-6 mo. rate | | | | | | |
| w(i) | 0.294 | 0.305 | 0.235 | 0.130 | 0.036 | 0.000 |
| StE(i) | 0.060 | 0.036 | 0.027 | 0.033 | 0.041 | 0.000 |
| t | 4.882 | 8.469 | 8.856 | 3.908 | 0.891 | 0.000 |

Table 11. Quarterly regression (1952-1970) of money on lagged income, lagged rate on commercial paper, price and trend

Lag polynomial degree: 3 — Number of iterations: 3

Before iterations: R-Bar Squared: 0.997 — Durbin-Watson Statistic: 0.3868

After iterations: R-Bar Squared: 0.999 — Durbin-Watson Statistic: 1.9675

Elasticities: $B_y = 0.851,\ (166.74)$ $B_{RCMP} = -.0770,\ (-5.130)$ $B_p = 0.491,\ (3.307)$ $B_t = -.0066,\ (-7.260)$

$B''_o = .0005;$ $\hat{p}_1 = 0.792,\ (11.473)$ $\hat{p}_2 = 0.596,\ (6.222)$ $\hat{p}_3 = 0.196\ (1.630)$

| Lagged independent variables | Quarters in the past 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| GNP | | | | | | |
| w(i) | 0.208 | 0.289 | 0.296 | 0.207 | 0.000 | |
| StE(i) | 0.060 | 0.044 | 0.030 | 0.051 | 0.000 | |
| t | 3.490 | 6.544 | 9.921 | 4.070 | 0.000 | |
| 4-6 mo. rate | | | | | | |
| w(i) | 0.283 | 0.322 | 0.248 | 0.126 | 0.021 | 0.000 |
| StE(i) | 0.057 | 0.037 | 0.025 | 0.033 | 0.041 | 0.000 |
| t | 5.011 | 8.761 | 9.938 | 3.830 | 0.519 | 0.000 |

Table 12. Quarterly regression (1952-1966) on real money on lagged real income, lagged rate on commercial paper, and trend

Lag polynomial degree: 3 | Number of iterations: 3

Before iterations: R-Bar Squared: 0.928 | Durbin-Watson Statistic: 0.3935

After iterations: R-Bar Squared: 0.999 | Durbin-Watson Statistic: 1.9976

Elasticities: $B_Y = 0.842$, (423.02) $B_{RCMP} = -.077$, (-7.430) $B_t = -.0053$ (-15.885)

$B''_o = .0020$; $\hat{p}_1 = 0.791$, (9.823) $\hat{p}_2 = 0.510$, (4.383) $\hat{p}_3 = .0059$ (0.443)

| Lagged independent variables | Quarters in the past 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| GNP | | | | | |
| w(i) | 0.257 | 0.271 | 0.271 | 0.201 | 0.000 |
| StE(i) | 0.059 | 0.051 | 0.029 | 0.052 | 0.000 |
| t | 4.362 | 5.322 | 9.214 | 3.833 | 0.000 |
| 4-6 mo. rate | | | | | |
| w(i) | 0.317 | 0.264 | 0.242 | 0.177 | 0.000 |
| StE(i) | 0.052 | 0.038 | 0.026 | 0.039 | 0.000 |
| t | 6.140 | 6.894 | 9.372 | 4.600 | 0.000 |

Table 13. Quarterly regression (1952-1966) of real money on lagged real income, lagged rate on commercial paper, and lagged rate on time deposits

Lag polynomial degree: 3 Number of iterations: 3

Before iterations: R-Bar Squared: 0.952 Durbin-Watson Statistic: 0.5030

After iterations: R-Bar Squared: 0.999 Durbin-Watson Statistic: 2.0028

Elasticities: $B_Y = 0.843$, (419.74) $B_{RCMP} = -.0755$, (-6.823) $B_{RTD} = -.224$, (-14.546)

$B_o'' = .0020$; $\hat{p}_1 = 0.733$, (8.145) $\hat{p}_2 = 0.682$, (6.109) $\hat{p}_3 = -.053$ (-0.391)

| Lagged independent variables | Quarters in the past 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| GNP | | | | | |
| w(i) | 0.225 | 0.270 | 0.287 | 0.217 | 0.000 |
| StE(i) | 0.059 | 0.050 | 0.029 | 0.051 | 0.000 |
| t | 3.818 | 5.364 | 9.743 | 4.221 | 0.000 |
| 4-6 mo. rate | | | | | |
| w(i) | 0.320 | 0.271 | 0.240 | 0.169 | 0.000 |
| StE(i) | 0.051 | 0.038 | 0.026 | 0.039 | 0.000 |
| t | 6.215 | 7.088 | 9.322 | 4.373 | 0.000 |
| Time deposit rate | | | | | |
| w(i) | 0.057 | 0.275 | 0.372 | 0.297 | 0.000 |
| StE(i) | 0.309 | 0.324 | 0.155 | 0.250 | 0.000 |
| t | 0.184 | 0.850 | 2.403 | 1.188 | 0.000 |

Table 14. Quarterly regression (1952-1966) of real money on lagged real income and lagged rate on commercial paper

Lag polynomial degree: 3 — Number of iterations: 3

Before iterations: R-Bar Squared: 0.429 — Durbin-Watson Statistic: 0.1151

After iterations: R-Bar Squared: 0.999 — Durbin-Watson Statistic: 2.0029

Elasticities: $B_Y = \underset{(266.19)}{0.843}$, $B_{RCMP} = \underset{(7.231)}{-.0830}$, $B''_o = -.0030$;

$\hat{p}_1 = \underset{(21.121)}{0.952}$, $\hat{p}_2 = \underset{(7.724)}{0.777}$, $\hat{p}_3 = \underset{(-0.879)}{-0.118}$

| Lagged independent variables | Quarters in the past 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| GNP | | | | | |
| w(i) | 0.235 | 0.271 | 0.283 | 0.212 | 0.000 |
| StE(i) | 0.056 | 0.048 | 0.028 | 0.049 | 0.000 |
| t | 4.194 | 5.639 | 10.097 | 4.347 | 0.000 |
| 4-6 mo. rate | | | | | |
| w(i) | 0.312 | 0.268 | 0.244 | 0.176 | 0.000 |
| StE(i) | 0.044 | 0.033 | 0.022 | 0.033 | 0.000 |
| t | 7.134 | 8.154 | 11.142 | 5.287 | 0.000 |

Table 15. Quarterly regression (1952-1966) of real money on lagged real income and lagged rate on treasury bills

Lag polynomial degree: 3 | Number of iterations: 2

<u>Before iterations</u>: R-Bar Squared: 0.829 | Durbin-Watson Statistic: 0.3034

<u>After iterations</u>: R-Bar Squared: 0.999 | Durbin-Watson Statistic: 1.7277

Elasticities: $B_Y = \underset{(103.136)}{0.877}$, $B_{R3TB} = \underset{(-5.577)}{-.279}$, $B''_o = -.0039$;

$\hat{p}_1 = \underset{(12.052)}{0.852}$, $\hat{p}_2 = \underset{(9.377)}{0.799}$

| Lagged independent variables | Quarters in the past 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| GNP | | | | | |
| w(i) | <u>0.261</u> | <u>0.275</u> | <u>0.270</u> | <u>0.194</u> | <u>0.000</u> |
| StE(i) | 0.077 | 0.066 | 0.039 | 0.071 | 0.000 |
| t | 3.378 | 4.169 | 6.993 | 2.732 | 0.000 |
| RTB | | | | | |
| w(i) | <u>0.201</u> | <u>0.287</u> | <u>0.300</u> | <u>0.212</u> | <u>0.000</u> |
| StE(i) | 0.064 | 0.049 | 0.032 | 0.056 | 0.000 |
| t | 3.138 | 5.825 | 9.375 | 3.779 | 0.000 |

Table 16. Quarterly regression (1952-1966) of real money on real income, rate on commercial paper, and the rate on time deposits

| | | | |
|---|---|---|---|
| | | Number of iterations: 2 | |
| <u>Before iterations</u>: | R-Bar Squared: 0.947 | Durbin-Watson Statistic: | 0.6843 |
| <u>After iterations</u>: | R-Bar Squared: 0.999 | Durbin-Watson Statistic: | 1.8749 |

Elasticities: $B_Y = \underset{(230.689)}{0.828}$, $B_{RCMP} = \underset{(-1.707)}{-.0173}$, $B_{RTD} = \underset{(-3.540)}{-0.267}$, $B''_o = .0022$;

$\hat{p}_1 = \underset{(6.281)}{0.634}$, $\hat{p}_2 = \underset{(15.198)}{0.900}$

Table 17. Quarterly regression (1952-1966) of real money on real income and the rate on commercial paper

---

Number of iterations: 2

| | | | | |
|---|---|---|---|---|
| <u>Before iterations</u>: | R-Bar Squared: | 0.275 | Durbin-Watson Statistic: | 0.0786 |
| <u>After iterations</u>: | R-Bar Squared: | 0.998 | Durbin-Watson Statistic: | 1.7778 |

Elasticities: $\underset{(149.105)}{B_Y = 0.834}$, $\underset{(-3.552)}{B_{RCMP} = -.0293}$, $B''_o = -.0068$;

$\underset{(25.986)}{\hat{p}_1 = 0.967}$, $\underset{(2.736)}{\hat{p}_2 = 0.353}$

---

Table 18. Annual regression (1954-1970) of money on lagged income, rate on commercial paper, and trend

Lag polynomial degree: 3 — Number of iterations: 2

Before iterations: R-Bar Squared: 0.988 — Durbin-Watson Statistic: 1.2877

After iterations: R-Bar Squared: 0.999 — Durbin-Watson Statistic: 1.6234

Elasticities: $B_y = 0.847$ (41.801), $B_{RCMP} = .0044$ (0.154), $B_t = -.0164$ (-4.830), $B''_o = -.0356$;

$\hat{p}_1 = 0.352$ (1.399), $\hat{p}_2 = 0.353$ (1.346)

| Lagged independent variables | Years in the past 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| GNP | | | | | |
| w(i) | 0.110 | 0.505 | 0.345 | 0.040 | 0.000 |
| StE(i) | 0.288 | 0.278 | 0.144 | 0.323 | 0.000 |
| t | 0.384 | 1.813 | 2.398 | 0.123 | 0.000 |

Table 19. Annual regression (1952-1966) of real money on real income, rate on commercial paper, and the rate on time deposits

| | | | |
|---|---|---|---|
| | | Number of iterations: 1 | |
| <u>Before iterations</u>: R-Bar Squared: | 0.810 | Durbin-Watson Statistic: | 1.0310 |
| <u>After iterations</u>: R-Bar Squared: | 0.999 | Durbin-Watson Statistic: | 1.5845 |

Elasticities: $B_Y = \underset{(90.628)}{0.841}$, $B_{RCMP} = \underset{(-3.810)}{-.0774}$, $B_{RTD} = \underset{(-10.373)}{-0.242}$, $B''_o = .0265$;

$\hat{p}_1 = \underset{(1.834)}{0.453}$

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Table 20. Annual regression (1954-1970) of money on income, rate on commercial paper, price and trend

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Number of iterations: 1

**Before iterations:** R-Bar Squared: 0.999 Durbin-Watson Statistic: 0.5865

**After iterations:** R-Bar Squared: 0.997 Durbin-Watson Statistic: 2.3096

Elasticities: $B_y = \underset{(50.006)}{0.946}$, $B_{RCMP} = \underset{(-2.758)}{-.0799}$, $B_p = \underset{(0.572)}{0.294}$, $B_t = \underset{(-1.245)}{-.0153}$

$B_o'' = -0.237$; $\hat{p}_1 = \underset{(3.291)}{0.537}$

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Table 21. Annual regression (1952-1966) of real money on real income and the rate on commercial paper

| | | |
|---|---|---|
| | | Number of iterations: 2 |
| Before iterations: | R-Bar Squared: 0.166 | Durbin-Watson Statistic: 0.5014 |
| After iterations: | R-Bar Squared: 0.999 | Durbin-Watson Statistic: 2.6799 |

Elasticities: $B_Y = \underset{(107.693)}{0.851}$, $B_{RCMP} = \underset{(-5.557)}{-0.103}$, $B''_o = -.0252$;

$\hat{p}_1 = \underset{(3.785)}{0.772}$, $\hat{p}_2 = \underset{(2.115)}{0.565}$

Table 22. Monthly regression (1952-1966) of real money on lagged real income, rate on commercial paper, and the rate on time deposits

Lag polynomial degree: 4 Number of iterations: 2

Before iterations: R-Bar Squared: 0.947 Durbin-Watson Statistic: 0.2399

After iterations: R-Bar Squared: 0.999 Durbin-Watson Statistic: 2.0611

Elasticities: $B_Y = \underset{(498.77)}{0.836}$, $B_{RCMP} = \underset{(-3.796)}{-.0180}$, $B_{RTD} = \underset{(-33.436)}{-0.268}$, $B''_O = .0031$;

$\hat{p}_1 = \underset{(24.347)}{0.877}$, $\hat{p}_2 = \underset{(4.130)}{0.299}$

| Lagged independent variables | Quarters in the past 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| GNP | | | | | | | | | | |
| w(i) | 0.230 | 0.180 | 0.140 | 0.110 | 0.090 | 0.079 | 0.071 | 0.060 | 0.040 | 0.000 |
| StE(i) | 0.048 | 0.036 | 0.033 | 0.029 | 0.033 | 0.032 | 0.025 | 0.032 | 0.036 | 0.000 |
| t | 4.767 | 4.968 | 4.192 | 3.750 | 2.722 | 2.489 | 2.804 | 1.897 | 1.113 | 0.000 |

Table 23. Monthly regression (1952-1966) of real money on lagged real income and the rate on commercial paper

Lag polynomial degree: 4 Number of iterations: 2

Before iterations: R-Bar Squared: 0.341 Durbin-Watson Statistic: 0.0367

After iterations: R-Bar Squared: 0.996 Durbin-Watson Statistic: 2.0473

Elasticities: $B_y = 0.834$, (201.877) $B_{RCMP} = -.0156$, (-3.074) $B''_o = -.0030$; $\hat{p}_1 = 0.987$, (68.395) $\hat{p}_2 = 0.287$ (3.972)

| Lagged independent variables | Quarters in the past 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| GNP | | | | | | | | | | |
| w(i) | 0.250 | 0.182 | 0.134 | 0.103 | 0.086 | 0.076 | 0.070 | 0.060 | 0.039 | 0.000 |
| StE(i) | 0.049 | 0.038 | 0.035 | 0.030 | 0.034 | 0.032 | 0.027 | 0.034 | 0.037 | 0.000 |
| t | 5.051 | 4.815 | 3.876 | 3.440 | 2.552 | 2.376 | 2.629 | 1.795 | 1.066 | 0.000 |

Table 24. Monthly regression (1952-1966) of real money on real income, rate on commercial paper, and the rate on time deposits

| | | | |
|---|---|---|---|
| | | | Number of iterations: 2 |
| Before iterations: | R-Bar Squared: 0.943 | | Durbin-Watson Statistic: 0.3445 |
| After iterations: | R-Bar Squared: 0.999 | | Durbin-Watson Statistic: 2.0272 |

Elasticities: $B_Y = 0.836,\ (338.202)$ $B_{RCMP} = -.0180,\ (-2.779)$ $B_{RTD} = -0.268,\ (-34.218)$ $B''_o = .0045;$

$\hat{p}_1 = 0.825,\ (19.430)$ $\hat{p}_2 = 0.138\ (1.830)$

Table 25. Monthly regression (1952-1966) of real money on real income and the rate on commercial paper

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Number of iterations: 2

| | | | | |
|---|---|---|---|---|
| <u>Before iterations:</u> | R-Bar Squared: | 0.285 | Durbin-Watson Statistic: | 0.0219 |
| <u>After iterations:</u> | R-Bar Squared: | 0.986 | Durbin-Watson Statistic: | 1.9926 |

Elasticities: $B_Y = \underset{(109.038)}{0.829}$, $B_{RCMP} = \underset{(-2.400)}{-.0152}$, $B''_O = -.0033$;

$\hat{p}_1 = \underset{(88.966)}{0.991}$, $\hat{p}_2 = \underset{(-0.386)}{-.0290}$

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